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Precision test of a Fermion mass texture

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Abstract

Texture zeros in the quark Yukawa matrices generally lead to precise and simple expressions for CKM matrix elements in terms of ratios of quark masses. Using the new data on b -decays we test a particularly promising texture zero solution and show that it is at best approximate. We analyse the approximate texture zero structure and show it is consistent with experiment. We investigate the implications for the CKM unitarity triangle, measurements at *BaBar* and *BELLE* as well as for the theories which invoke family symmetries.

1 Introduction

The structure of quark and lepton mass matrices provides us with a rare insight into the physics beyond the Standard Model which may directly probe the underlying theory at the gauge unification or Planck scale. While the quark mass matrices and the CKM matrix, V^{CKM} , are intimately related, measurement of the eigenvalues of the mass matrices and the matrix elements of V^{CKM} is not sufficient to determine the structure of the full mass matrix and of the matrix of Yukawa couplings giving rise to them. Given this under-determination, the phenomenological approach most often used is to make some assumption about this structure and explore the experimental consequences for the V_{ij}^{CKM} . A particularly promising starting point assumes that there are anomalously small entries in the up and down quark Yukawa

matrices - “texture zeros”¹. These lead to relations for the V_{ij}^{CKM} in terms of ratios of quark masses which do not involve any unknown couplings and hence can be precisely tested. Various texture zeros have been studied. For the case of symmetric mass matrices a systematic analyses determining which combinations of textures involving 4, 5 or 6 zeros for the U , D matrices are compatible with data was carried out in [1]. The main reason for looking for such texture zero solutions is that they may shed light on physics beyond the Standard Model, for example the presence of a new family symmetry relating different generations.

The new generation of b-factory experiments has led to more precise measurements of the CKM matrix elements that, together with the progress in understanding hadronic uncertainties [2] and light quark masses, allows us to test texture zero structures to a greater precision than has hitherto been possible. In this paper we will study the most promising structure based on simultaneous zeros in the U and D mass matrices at the (1,1) and (1,3) positions. In the additional hypothesis of equal magnitude of the (1,2) and (2,1) entries and of sufficiently small (3,2) entry, this structure gives three precise relations between V_{ij}^{CKM} and ratios of quark masses, leaving only one CKM element undetermined. We find that the new experimental and theoretical information suggests that (at least) one of the hypothesis leading to those precise texture zero relations needs to be relaxed. The simplest possibility is that while the (1,3) element is small it is *non-* zero so that the texture zero is only approximate. As a result one of the relations, the texture zero prediction for $|V_{ub}/V_{cb}|$, is modified, as is suggested by the new precise data; the other two relations are less affected.

In the context of an underlying family symmetry this result is to be expected for the family symmetry usually requires texture zeros to be only approximate and, in some cases, actually predicts the order at which the approximate zero should be filled in. For example, a very simple Abelian family symmetry predicts the (1,3) element should be nonzero at a level consistent with the new data while requiring the (1,1) element should be much smaller, preserving the remaining two texture zero predictions.

Another possibility [3] is that the (3,2) entry in the down quark mass matrix is not as small as the (2,3) entry (barring cancellations, the latter must be smaller than the (3,3) entry by a factor $\mathcal{O}(|V_{cb}|)$). Such an asymmetry can also be easily achieved in the context of Abelian or non-Abelian family symmetries and offers an intriguing connection with neutrino physics. A sizeable (3,2) entry in the down quark mass matrix is in fact a generic prediction of a class of unified models of quark and lepton masses and mixings. In such models, a large leptonic mixing angle accounting for the atmospheric neutrino anomaly originates from a sizeable (2,3) entry in the charged lepton mass matrix. The GUT symmetry then forces the (3,2) element in the down quark matrix to be of the same order of magnitude [4]. However we note that the original symmetric structure can also lead, very naturally, to a large neutrino mixing angle [5]. Finally, the third situation leading to a correction to the texture zero relations is that the entries

¹Strictly texture zeros can only apply at a single mass scale (the GUT or string scale?) and will be filled in by Renormalisation Group running. However, in general, such effects are very small and the texture zeros persist to a good approximation at all scales.

(1,2) and (2,1) in the up quark mass matrix are not equal in magnitude. This can happen in non unified abelian models due to different order one coefficients. We will not investigate this possibility in this paper because it destroys one of the successful texture zero predictions (see the discussion in Section 2 below). From this one sees that detailed tests of the texture zero relations will help to identify the underlying family symmetry.

Our analysis does not take into account possible new physics contributions to the processes constraining the CKM parameters. Such contributions might affect the experimental determination of $|V_{td}/V_{ts}|$ through modifications to B mixing and the CP -violating part of K mixing (or both) and consequently affect the corresponding texture zero relation. However we do not expect $|V_{ub}/V_{cb}|$ to be similarly affected where the main uncertainty instead lies in the hadronic modelling of charmless semileptonic B decays. Our analysis addresses the phenomenological problem of the $|V_{ub}/V_{cb}|$ prediction of texture zero structures and its remedies and, as a consequence, most of our analysis would not be affected by new physics effects. Very recent measurements of $\sin 2\beta$ from *BaBar* and *BELLE* [6] do raise the possibility of beyond Standard Model contributions to CP -violation and therefore the quantitative fits performed in Sections 2 and 3 might be affected by supersymmetric contributions to K and B mixing. We briefly discuss how low values of $\sin 2\beta$ affect our analysis.

The paper is structured as follows. In Section 2 we present the experimental tests of the texture zero predictions following from zeros in the U and D mass matrices at the (1,1) and (1,3) positions. In Section 3 we discuss the implications for the mass matrices following from the need to modify one of the texture zero relations. We develop a perturbative expansion which allows us to identify the possible corrections to the texture zero predictions. In Section 4 we consider the implications of such structure for an underlying family symmetry and finally in Section 5 we present conclusions.

2 Tests of texture zero predictions

In what follows we assume that the off-diagonal entries are small relative to their on-diagonal partners so that one may develop a perturbative expansion for the CKM matrix elements [7, 8, 9, 10]. This is a reasonable starting point because it immediately leads to small mixing angles consistent with observation. We further assume here that there are texture zeros in the (1,1) and (1,3) elements, that the (1,2) and (2,1) elements have equal magnitude and that the texture is approximately symmetric ((3,2) \sim (2,3)). These assumptions lead to the texture zero relations [1, 8, 11, 10, 12]

$$\left| \frac{V_{ub}}{V_{cb}} \right| = \sqrt{\frac{m_u}{m_c}} \quad \left| \frac{V_{td}}{V_{ts}} \right| = \sqrt{\frac{m_d}{m_s}} \quad (1)$$

$$|V_{us}| = \lambda = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\phi} \sqrt{\frac{m_u}{m_c}} \right| \quad (2)$$

Fixed Parameters		
Parameter	Value	Reference
G_F	$1.16639 \times 10^{-5} \text{GeV}^{-2}$	[15]
M_W	$(80.42 \pm 0.06) \text{GeV}$	[15]
f_K	$(0.161 \pm 0.0015) \text{GeV}$	[15]
m_K	$(0.497672 \pm 0.000031) \text{GeV}$	[15]
Δm_K	$(3.491 \pm 0.009) \times 10^{15} \text{GeV}$	[15]
$ \epsilon_K $	$(2.271 \pm 0.017) \times 10^{-3}$	[15]
η_2^*	(0.574 ± 0.004)	[16]
m_{B_d}	$(5.2792 \pm 0.0018) \text{GeV}$	[15]
η_B	0.55 ± 0.01	[16]
m_{B_s}	$(5.3693 \pm 0.0020) \text{GeV}$	[15]

Table 1: Fixed Parameters.

We will prove that, quite generally, ϕ is approximately the Standard Model CP violating phase (for more restricted cases see [13, 14]).

Barbieri et al (BHR) [11] emphasized that these relations lead to a very tight determination of the CKM unitarity triangle. In terms of the re-scaled Wolfenstein parameters, $\bar{\rho} = c\rho$, $\bar{\eta} = c\eta$, $c = (1 - \lambda^2/2)$, the ratios $|V_{ub}|/|V_{cb}|$ and $|V_{td}|/|V_{ts}|$ are

$$\frac{|V_{ub}|}{|V_{cb}|} = \frac{\lambda}{c} \sqrt{\bar{\rho}^2 + \bar{\eta}^2} \quad \frac{|V_{td}|}{|V_{ts}|} = \frac{\lambda}{c} \sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2} \quad (3)$$

In the context of the Standard Model, the measurable quantities which give information on $\bar{\rho}$ and $\bar{\eta}$ are

- (a) the ratio $|V_{ub}|/|V_{cb}|$ obtained from semi-leptonic decays of B mesons,
- (b) Δm_{B_d} and Δm_{B_s} which are the mass differences in the $B_d^0 - \bar{B}_d^0$, $B_s^0 - \bar{B}_s^0$ systems,
- (c) $|\epsilon_K|$ the parameter related to CP violation in the K , \bar{K} system, and
- (d) $\sin 2\beta$ (β is one of the angles in the unitarity triangle) obtained from CP asymmetries in various B decays.

2.1 Standard Model (SM) fit

In comparing the texture zeros with experiment we proceed in two stages. We first use the latest data to find λ , $\bar{\rho}$ and $\bar{\eta}$ and then compare the result with eq(3). Our procedure is to construct a two-dimensional probability density for $\bar{\rho}$ and $\bar{\eta}$ [17] from the constraints of the above measurements. Entering in the fits are several parameters which have been well measured and which we choose not to vary. These are given in Table 1. At present we have only an upper

limit for Δm_{B_s} and we employ the so-called ‘amplitude method’ to include this information into the fit[18].

Our fit assumes the Standard Model (SM) relations between the experimental measurables and the CKM matrix elements. In a supersymmetric extension of the Standard Model we expect that there will be corrections to these measurables; $|\epsilon_K|$ is particularly sensitive to such corrections. To allow for this possibility we carry out a separate fit in which the data on $|\epsilon_K|$, together with that on $\sin 2\beta$ for which there are rather different measurements, is dropped. The mass differences Δm_{B_d} , Δm_{B_s} and $\sin 2\beta$ might also be affected by supersymmetry, especially if the structure of squark mass matrices is determined by the flavour symmetry accounting for quark masses and mixings. We do not consider this possibility here.

The formulas used in the standard model fit are

$$\Delta m_{B_d} = C_{\Delta m_{B_d}} A^2 \lambda^6 [(1 - \bar{\rho})^2 + \bar{\eta}^2] m_{B_d} f_{B_d}^2 B_{B_d} \eta_{B_d} S(x_t^*), \quad (4)$$

where $C_{\Delta m_{B_d}} = \frac{G_F^2 M_W^2}{6\pi^2}$. Here $S(x_t^*)$ is the standard Inami-Lim function [19] and $f_B^2 B_B$ is the product of the B meson decay constant and the B parameter analogous to B_K in the K system.

$$\Delta m_{B_s} = \Delta m_{B_d} \frac{m_{B_s}}{m_{B_d}} \xi^2 \frac{c^2}{\lambda^2} \frac{1}{(1 - \bar{\rho})^2 + \bar{\eta}^2}, \quad (5)$$

where

$$\xi = \frac{f_{B_s} \sqrt{B_{B_s}}}{f_{B_d} \sqrt{B_{B_d}}}. \quad (6)$$

For the ϵ_K parameter we have

$$\begin{aligned} |\epsilon_K| &= C_\epsilon B_K A^2 \lambda^6 \bar{\eta} \left[-\eta_1^* x_c^* + A^2 \lambda^4 \left(1 - \bar{\rho} - (\bar{\rho}^2 + \bar{\eta}^2 - \bar{\rho}) \lambda^2 \right) \eta_2^* S(x_t^*) \right. \\ &\quad \left. + \eta_3^* S(x_c^*, x_t^*) \right], \end{aligned} \quad (7)$$

where $C_\epsilon = \frac{G_F^2 f_K^2 m_K m_W^2}{6\sqrt{2}\pi^2 \Delta m_K}$. The short distance QCD corrections are contained in the coefficients η_i^* , which have been computed at the Next to Leading-logarithmic Order (NLO) in [20]. The NLO calculation requires the use of the one-loop relation between the pole mass and the running mass $m_i^{pole} = m_i^* \left(1 + \frac{\alpha_s(m^*)}{\pi} \frac{4}{3} \right)$ in the $\overline{\text{MS}}$. The coefficients η_i^* have been evaluated at $\Lambda_{\overline{\text{MS}}}^{NLO} = 371 \text{ MeV}$. All the starred quantities in Table 2 are given in terms of these quantities, for example $x_i^* = [m_i^*]^2 / M_W^2$.

Finally the angles of the unitarity triangle are given by

$$\begin{aligned} \sin 2\beta &= \frac{2\bar{\eta}(1 - \bar{\rho})}{\bar{\eta}^2 + (1 - \bar{\rho})^2} \\ \sin 2\alpha &= \frac{2\bar{\eta}(\bar{\eta}^2 + \bar{\rho}(\bar{\rho} - 1))}{(\bar{\eta}^2 + (1 - \bar{\rho})^2)(\bar{\eta}^2 + \bar{\rho}^2)} \\ \sin 2\gamma &= \frac{2\bar{\rho}\bar{\eta}}{\bar{\rho}^2 + \bar{\eta}^2} \end{aligned} \quad (8)$$

Fitted Parameters			
Parameter	Value	Gaussian-Flat errors.	Referen.
A	0.834 ± 0.036		*
λ	0.2196 ± 0.0023		*
$ V_{ub} ^{\text{CLEO}}$	32.5×10^{-4}	$(\pm 2.9 \pm 5.5) \times 10^{-4}$	[21]
$ V_{ub} ^{\text{LEP}}$	41.3×10^{-4}	$(\pm 6.3 \pm 3.1) \times 10^{-4}$	[22]
$ V_{cb} $	$(41.0 \pm 1.6) \times 10^{-3}$		[2]
B_K	0.87 ± 0.143	0.06 ± 0.13	[16]
m_c^*	$(1.3 \pm 0.1)\text{GeV}$		*
m_t^*	$(167 \pm 5)\text{GeV}$		*
η_1^*	1.38 ± 0.53		[16]
η_3^*	0.47 ± 0.04		[16]
Δm_{B_d}	$(0.487 \pm 0.014)\text{ps}^{-1}$		[23]
$f_{B_d}\sqrt{B_{B_d}}$	$(0.230 \pm 0.032)\text{GeV}$	$\pm 0.025\text{--}\pm 0.020$	[2]
ξ	(1.14 ± 0.064)	0.04 ± 0.05	[2]
$\Delta m_{B_s} >$	15ps^{-1} at 95% C.L.		[23]
$\sin 2\beta$	0.41 ± 0.17		[6]
$\sin 2\beta$	0.47 ± 0.16		[24]

Table 2: Fitted Parameters. The parameters marked with * have been computed here with the new data from [15].

Using these expressions we carried out fits to the parameters listed in Table 2 keeping the well determined parameters listed in Table 1 fixed.

The results in the $\bar{\rho} - \bar{\eta}$ plane of the fit are shown in Fig.1. The confidence limits shown correspond to 68%, 95% and 99%. In the experimental fits (which assume the SM) the results of $|V_{ub}/V_{cb}|$ for both *CLEO* and *LEP* collaborations (see Table 2) were included. Although the result from the *CLEO* collaboration is lower than the result from *LEP*, they are consistent within one σ . The combination of both, assuming a Gaussian distribution for the experimental errors and a flat distribution for the theoretical errors, gives a value of 0.087 ± 0.010 , which is consistent with the PDG value of 0.090 ± 0.025 [15].

As can be seen from Fig 1 the constraints on $\bar{\rho}$ and $\bar{\eta}$ would be considerably strengthened by a measurement of Δm_{B_s} , which presently has only a lower limit, and by an improvement in the precision on $\sin 2\beta$. We can see from Fig.1 this agrees within 2σ with the the new value for the parameter $\sin 2\beta = 0.47 \pm 0.16$ (combining *BaBar* and *Belle* results with those from *CDF* and *ALEPH* [24]). However there is a deviation at the 1σ level which may be a hint for physics beyond the SM.

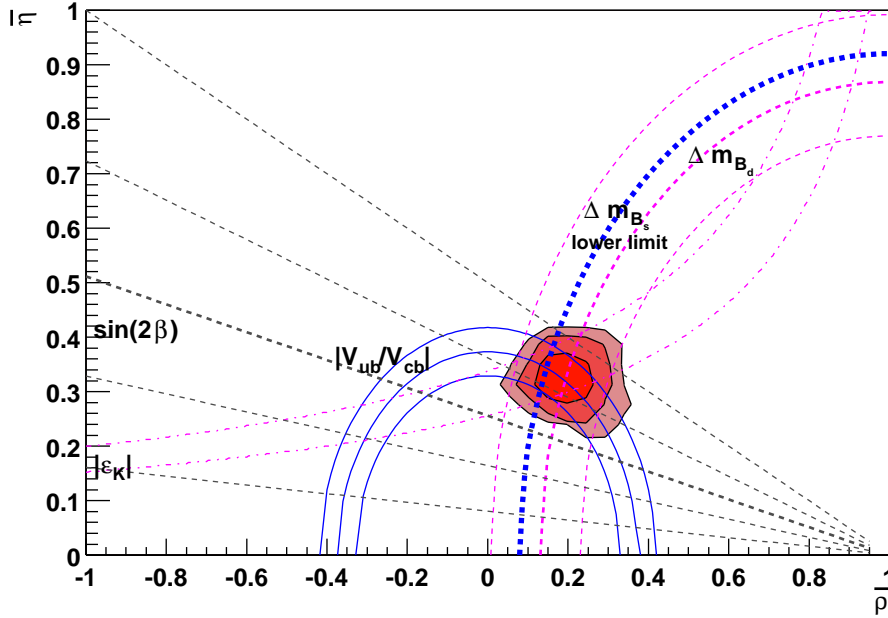


Figure 1: The SM fit of Section 2.1 to $|V_{ub}/V_{cb}|$, Δm_{B_s} (lower limit), Δm_{B_d} , $|\epsilon_K|$ and the recent result for $\sin 2\beta$. The lines indicate the region of 1σ and, in the case of $\sin 2\beta$, also the 2σ region, demonstrating that the new value is still consistent with the rest of SM constraints within 2σ . The CL are at 99%, 95% and 68%.

2.2 Comparison with the texture zero predictions.

We are now able to compare the experimental results with the texture zero predictions of eq(1). In Fig.2 we show the region in the $\bar{\rho} - \bar{\eta}$ plane allowed by these relations together with the various constraints following from the processes (a)-(d). In this fit we have taken symmetric forms for the U and D Yukawa matrices with texture zeros in the (1,1), (1,3) and (3,1) positions. Comparison with Fig 1 show that the predictions are hard to reconcile with the data, being consistent only at greater than the 99% CL (for the case $|\epsilon_K|$ and $\sin 2\beta$ are not included). Fig.2 shows that one of the reasons for the poor agreement is the measurement of $|V_{ub}/V_{cb}|$. Given the fact the CLEO and LEP measurements differ considerably it is of interest to consider whether the discrepancy disappears if we use only the value for the *CLEO* collaboration. In fact we find that this only marginally changes things. One may see that the significant improvement in the experimental measurements, particularly of $|V_{ub}/V_{cb}|$ and the improved lower limit on Δm_{B_s} , strongly disfavour this promising texture zero scheme. As remarked before, this conclusion holds in extensions of the SM too, since new physics effects might affect the mass difference Δm_{B_s} , but cannot alleviate the disagreement with V_{ub}/V_{cb} .

Given this discrepancy, do we have to abandon the texture zero solution completely? In fact we do not, as we now show. The problematic relation $|V_{ub}/V_{cb}| = \sqrt{m_u/m_c}$ (and, to a lesser extent, the other two texture zero predictions) depends on three assumptions.

- Texture zeros: the matrix elements Y_{13} , Y_{31} , Y_{11} are negligibly small both in the up

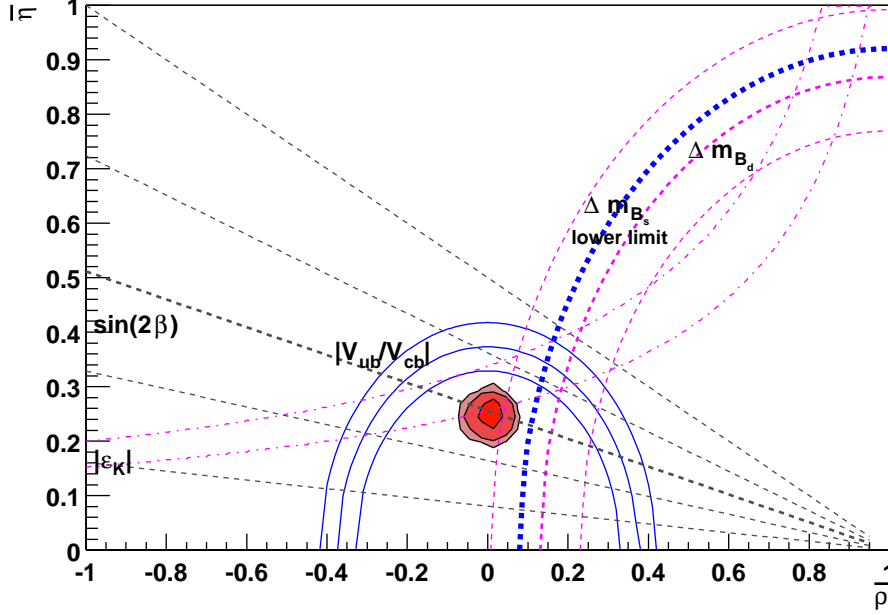


Figure 2: The fit for the relations $|V_{td}|/|V_{ts}| = \sqrt{m_d/m_s}$ and $|V_{ub}|/|V_{cb}| = \sqrt{m_u/m_c}$. The experimental constraints + SM interpretation indicate that these relations are disfavoured.

($Y = U$) and down ($Y = D$) sector. Actually, as mentioned above, the underlying theory generating the texture zero is unlikely to guarantee a particular mass matrix element is absolutely zero. Moreover an exact zero can only apply at a single mass scale for radiative effects necessarily generate contributions to all the elements of the quark mass matrices.

- Small higher order corrections in the perturbative diagonalization of the up and down quark mass matrices. The expected correction is of order 7% or less if $|D_{32}| \sim |V_{cb}||D_{33}|$ and becomes important for larger values of $|D_{32}|$. Larger values of $|D_{32}|$ can arise in unified models in which a large leptonic mixing originates from the charged lepton sector. They also arise in models with a texture zero in the (2,2) position and no cancellation between down and up quark contributions to V_{cb} . In this case one has in fact $|D_{23}/D_{33}| \sim |V_{cb}|$. As a consequence, an asymmetry $|D_{32}/D_{33}| > |D_{23}/D_{33}|$ is required in order to account for the value of $m_s/m_b = |D_{23}/D_{33} \cdot D_{32}/D_{33}|$.
- $|U_{12}| = |U_{21}|$. This condition is usually met in unified models. In the case of SU(5) models for example, U_{12} and U_{21} are both generated by operators that can be written in the form $\langle \phi \rangle T_1 T_2 H$, where $T_{1,2}$ are the tenplets of the first and second family containing the up quarks, H is the up Higgs fiveplet and ϕ represents a (normalized) set of fields whose vev is SM invariant. The relation $|U_{12}| = |U_{21}|$ follows unless $\langle \phi \rangle$ breaks SU(5). In the latter case, SU(5) Clebsch coefficients will differentiate $|U_{12}|$ and $|U_{21}|$, but often in a too violent way, leading to an even worse disagreement with the V_{ub}/V_{cb} prediction. Moreover, the analogous operator in the down quark and charged lepton sector would spoil the successful relation $m_d m_s / m_b^2 \sim m_e m_\mu / m_\tau^2$. The condition $|U_{12}| = |U_{21}|$ is automatically met in some

non-Abelian models [27, 26]. In any case we regard the phenomenological success of the relation for $|V_{us}|$ given by eq(2) as a result that should be preserved.

In what follows we focus on the possibility that either the first or the second assumption is not fulfilled. In the first case we still assume a symmetric structure for U and D and since in the second case D is manifestly asymmetric, we refer to these two scenarios as the *symmetric* and *asymmetric* texture cases.

In particular, in Section 3.2 we study in detail textures with small but non negligible (1,3) element. As we discuss there, the order at which this element arises is a characteristic prediction of a family symmetry so determination of this element is a discriminator between various candidate family symmetries. Moreover we show that, even if we drop the constraint of an exact texture zero in the (1,3) position, two of the three texture zero predictions remain and are in good agreement with the data. Finally we prove that the identification of the phase ϕ in the expression for V_{us} , eq(2), is true in leading order even after allowing for a matrix element in the (1,3) position. In 3.3 we consider the complementary possibility of an asymmetric texture.

3 Non-zero s_{13} : Perturbative Analysis

In this Section we use the notation of Hall and Rasin [8]. We start with the Yukawa matrices \mathbf{Y} ($\mathbf{Y}=\mathbf{U}$ or \mathbf{D}) with the assumption that the entries in the Yukawa matrices have a hierarchical structure, with Y_{33} being the largest. For the purposes of explaining the important aspects of the analysis it is useful to first take Y_{ij} to be real and later consider how the analysis is modified by CP violating phases. The matrices \mathbf{Y} can be diagonalized by three successive rotations in the (2,3), (1,3) and (1,2) sectors (denoted by s_{23}, s_{13} and s_{12}):

$$\begin{pmatrix} \tilde{Y}_{11} & 0 & 0 \\ 0 & \tilde{Y}_{22} & 0 \\ 0 & 0 & \tilde{Y}_{33} \end{pmatrix} = \begin{pmatrix} 1 & -s_{12}^Y & 0 \\ s_{12}^Y & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -s_{13}^Y \\ 0 & 1 & 0 \\ s_{13}^Y & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -s_{23}^Y \\ 0 & s_{23}^Y & 1 \end{pmatrix} \times \\ \times \begin{pmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & s_{23}'^Y \\ 0 & -s_{23}'^Y & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & s_{13}'^Y \\ 0 & 1 & 0 \\ -s_{13}'^Y & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & s_{12}'^Y & 0 \\ -s_{12}'^Y & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (9)$$

In terms of these angles the CKM matrix is given by

$$\mathbf{V} = \begin{pmatrix} 1 & s_{12} + s_{13}^U s_{23} & s_{13} - s_{12}^U s_{23} \\ -s_{12} - s_{13}^D s_{23} & 1 & s_{23} + s_{12}^U s_{13} \\ -s_{13} + s_{12}^D s_{23} & -s_{23} - s_{12}^D s_{13} & 1 \end{pmatrix}, \quad (10)$$

where $s_{23} = s_{23}^D - s_{23}^U$, $s_{13} = s_{13}^D - s_{13}^U$ and $s_{12} = s_{12}^D - s_{12}^U$.

It is straightforward now to see the origin of the texture zero relations eq(1). From eq(10) we see it is sufficient to have [8]:

- $\frac{|V_{ub}|}{|V_{cb}|} = |s_{12}^U|$ and $\frac{|V_{td}|}{|V_{ts}|} = |s_{12}^D|$ which is obtained by:

$$|s_{13}| \ll |s_{12}^U s_{23}| \text{ and } |s_{13}| \ll |s_{12}^D s_{23}|. \quad (11)$$

This condition will define how small the (1,3) element must be in the mass matrices to obtain the (1,3) “texture zero” prediction. Notice that since the “13” rotations are performed after the “23” rotations, what determines the size of the rotation s_{13} is the “effective” element \tilde{Y}_{13} in eq. (16). As eq. (16) shows, \tilde{Y}_{13} depends not only on Y_{13} but also on the size of the Y_{21} element, which is rotated in the (3,1) position by the right-handed quark rotation $s_{23}'^Y$. The size of the Y_{21} element can be related to the light quark masses, so that the conditions (11) also defines how small the right-handed rotation $s_{23}'^Y$ must be in order to obtain the texture zero prediction.

In addition we require

- $|s_{12}^U| = \sqrt{\frac{m_u}{m_c}}$ and $|s_{12}^D| = \sqrt{\frac{m_d}{m_s}}$ which is obtained by:

$$|\tilde{Y}_{11}| \ll \left| \frac{\tilde{Y}_{12}\tilde{Y}_{21}}{\tilde{Y}_{22}} \right| \text{ and } |\tilde{Y}_{12}| = |\tilde{Y}_{21}|. \quad (12)$$

This is the (1,1) texture zero condition together with the symmetry needed to obtain eq(1,2). To proceed further we need to determine the mixing angles in terms of the Yukawa couplings. To do this we assume the off diagonal elements are small relative to the on-diagonal ones in each step of the diagonalisation, leading to the perturbative relation for the small mixing angles given by²

$$\begin{aligned} s_{23}^Y &\simeq \frac{Y_{23}}{Y_{33}} + \frac{Y_{32}Y_{22}}{Y_{33}^2}, \quad s_{23}'^Y \simeq \frac{Y_{32}}{Y_{33}} + \frac{Y_{23}Y_{22}}{Y_{33}^2} \\ s_{13}^Y &\simeq \frac{\tilde{Y}_{13}}{Y_{33}} + \frac{\tilde{Y}_{31}Y_{11}}{Y_{33}^2}, \quad s_{13}'^Y \simeq \frac{\tilde{Y}_{31}}{Y_{33}} + \frac{\tilde{Y}_{13}Y_{11}}{Y_{33}^2} \\ s_{12}^Y &\simeq \frac{\tilde{Y}_{12}}{\tilde{Y}_{22}} + \frac{\tilde{Y}_{21}Y_{11}}{\tilde{Y}_{22}^2}, \quad s_{12}'^Y \simeq \frac{\tilde{Y}_{21}}{\tilde{Y}_{22}} + \frac{\tilde{Y}_{12}Y_{11}}{\tilde{Y}_{22}^2} \end{aligned} \quad (13)$$

The successive rotations produce elements

$$\tilde{\tilde{Y}}_{11} \simeq \tilde{Y}_{11} - \frac{\tilde{Y}_{12}\tilde{Y}_{21}}{\tilde{Y}_{22}}, \quad \tilde{Y}_{11} \simeq Y_{11} - \frac{\tilde{Y}_{13}\tilde{Y}_{31}}{Y_{33}}, \quad \tilde{Y}_{22} \simeq Y_{22} - \frac{Y_{23}Y_{32}}{Y_{33}}, \quad (14)$$

and

$$\tilde{Y}_{12} = Y_{12} - Y_{13}s_{23}'^Y, \quad \tilde{Y}_{21} = Y_{21} - Y_{31}s_{23}^Y, \quad (15)$$

$$\tilde{Y}_{13} = Y_{13} + Y_{12}s_{23}'^Y, \quad \tilde{Y}_{31} = Y_{31} + Y_{21}s_{23}^Y. \quad (16)$$

²Actually this equation defines just how small the off diagonal elements need be since successive terms in the expansion should be well ordered.

From this equation one may see that the contribution of terms involving elements below the diagonal are suppressed by inverse powers of the heavier quark masses. For this reason they are only weakly constrained by the CKM mixing angles, with the only possible exception of the (3,2) element [3]. Returning to the condition eq(11) for the (1,3) texture zero prediction we see that Y_{13} must be small. The condition on Y_{31} is much weaker due to the heavy quark suppression. As we shall discuss the larger hierarchy in the up quark masses means that the down quark contribution to the mixing angles dominates. Thus, to a good approximation the requirement for the (1,3) texture zero prediction is

$$\tilde{D}_{13} \ll \frac{\tilde{U}_{12} D_{23}}{\tilde{U}_{22}} \quad (17)$$

We are interested in what happens if this condition is *not* satisfied. In this case, from eqs(10) and (11), we have

$$\begin{aligned} \frac{|V_{ub}|}{|V_{cb}|} &\approx \left| \sqrt{\frac{m_u}{m_c}} - \frac{s_{13}}{s_{23}} \right| \\ \frac{|V_{td}|}{|V_{ts}|} &\approx \left| \sqrt{\frac{m_d}{m_s}} - \frac{s_{13}}{s_{23}} \right| \end{aligned} \quad (18)$$

How is this analysis affected if Y_{ij} are complex? We discuss this in detail in the next Section but the implications are easy to anticipate. The sequence of rotations in eq(9) will now be interspersed with various diagonal rephasing matrices. This will change the above equations introducing phases in various terms but cannot induce any new terms in V_{ij} . However, it is clear that even in this case eqs(11) and (12) *are* the correct conditions for yielding the predictions eq(1). In turn this means that eq(18) remains correct although the terms proportional to $\frac{s_{13}}{s_{23}}$ may acquire different phases in the two equations (see below). However this does not change the conclusion about which texture zero prediction receives the dominant correction.

Note that the texture zero predictions are modified in a definite way in that the prediction for $|V_{ub}|/|V_{cb}|$ in general has a larger percentage change than that for $|V_{td}|/|V_{ts}|$ because the mass hierarchy for the up quarks is larger than that for the down quarks while the correction proportional to s_{13}/s_{23} remains the same in both cases. This is just what is needed to correct the disagreement we found when comparing experiment with the texture zero prediction. The correction term to the ratio $|V_{ub}|/|V_{cb}|$ is

$$1 - c_U \sin \psi + \frac{1}{2} c_U^2 \quad (19)$$

where $c_U = \frac{s_{13}}{s_{23}} / \sqrt{\frac{m_u}{m_c}}$ and ψ is the relative phase between the two terms when the CP phase angle ϕ is 90° while the correction term to the ratio $|V_{td}|/|V_{ts}|$ is

$$1 - c_D \cos \psi + \frac{1}{2} c_D^2 \quad (20)$$

where $c_D = \frac{s_{13}}{s_{23}} / \sqrt{\frac{m_d}{m_s}}$. If we take $\frac{|D_{13}|}{|D_{23}|} \approx 0.04$ and $\psi \approx -45^\circ$ then we can easily get a 90% increase, for example, to $|V_{ub}|/|V_{cb}|$ while affecting $|V_{td}|/|V_{ts}|$ by only about 10%.

3.1 Inclusion of phases - General parametrization

Consider the Yukawa matrices for the case where U_{11} and D_{11} are both zero, i.e. just one texture zero in each. From the arguments of Kosenko and Shrock [28] there would be then be a total of eight unremovable phases. We may assign the eight phases as $\phi_{12}^{U,D}$, $\phi_{13}^{U,D}$, $\phi_{22}^{U,D}$ and $\phi_{23}^{U,D}$ so that all possible quantities which are invariant under re-phasing transformations, e.g. $U_{12}D_{23}U_{22}^*D_{13}^*$ are not, in general, real – in accordance with the discussion of Kosenko and Shrock. Having so many phases is unnecessary as far as the physics is concerned since the CKM matrix elements computed from eq(10) in the heavy quark limit depend on only two independent combinations of the eight phases. Using eqs(13-16) we have

$$V_{us} = \frac{|\tilde{D}_{12}|}{|\tilde{D}_{22}|} e^{i(\phi_{12}^D - \phi_{22}^D)} - \frac{|\tilde{U}_{12}|}{|\tilde{U}_{22}|} e^{i(\phi_{12}^U - \phi_{22}^U)} \quad (21)$$

i.e. $|V_{us}|$ depends on the phase $\phi_1 = (\phi_{12}^U - \phi_{22}^U) - (\phi_{12}^D - \phi_{22}^D)$. In addition we have

$$\begin{aligned} V_{ub} &= \frac{|\tilde{D}_{13}|}{|\tilde{D}_{33}|} e^{i\phi_{13}^D} - \frac{|\tilde{U}_{12}|}{|\tilde{U}_{22}|} e^{i(\phi_{12}^U - \phi_{22}^U)} \frac{|\tilde{D}_{23}|}{|\tilde{D}_{33}|} e^{i\phi_{23}^D} \\ V_{td} &= -\frac{|\tilde{D}_{13}|}{|\tilde{D}_{33}|} e^{i\phi_{13}^D} + \frac{|\tilde{D}_{12}|}{|\tilde{D}_{22}|} e^{i(\phi_{12}^D - \phi_{22}^D)} \frac{|\tilde{D}_{23}|}{|\tilde{D}_{33}|} e^{i\phi_{23}^D} \end{aligned} \quad (22)$$

Thus the magnitudes $|V_{ub}|$, $|V_{td}|$ depend on only the combinations $\phi_2 = (\phi_{13}^D - \phi_{23}^D) - (\phi_{12}^D - \phi_{22}^D)$ and $\phi_1 - \phi_2$ respectively. As a result so must the Wolfenstein parameters ρ and η depend only on the two phases ϕ_1 and ϕ_2 . Moreover, evaluating the invariant $J = \text{Im}\{V_{cb}V_{us}V_{cs}^*V_{ub}^*\}$ which determines the magnitude of CP violation, we find

$$\eta \propto \text{Im}[J] = \frac{|\tilde{D}_{23}|}{|\tilde{D}_{33}|} \left[\frac{|\tilde{D}_{23}|}{|\tilde{D}_{33}|} \frac{|\tilde{D}_{12}|}{|\tilde{D}_{22}|} \frac{|\tilde{U}_{12}|}{|\tilde{U}_{22}|} \sin \phi_1 - \frac{|\tilde{D}_{13}|}{|\tilde{D}_{33}|} \left(\frac{|\tilde{D}_{12}|}{|\tilde{D}_{22}|} \sin \phi_2 + \frac{|\tilde{D}_{12}|}{|\tilde{U}_{22}|} \sin(\phi_1 - \phi_2) \right) \right] \quad (23)$$

For small $|D_{13}|$, the first term is the leading one and if the sub-leading corrections were negligible, then the CP violating phase ϕ_{CP} would be just ϕ_1 , the *same* phase which enters in eq(2). In this case the phase ϕ_1 is simply related to the ‘standard’ (i.e. PDG convention) of the CP -violating phase, δ by $\delta = \pi - \phi_1 - \beta$ where β is the angle appearing in the unitarity triangle. The next leading correction is the second term, proportional to $\sin \phi_2$. Our proof that the phase ϕ_1 which drives the CP -violating phase is the same one that appears in eq(2) follows simply from the suppression of terms in the heavy quark limit and the equality of the (1, 2) and (2, 1) elements. This generalizes the previous proof of this result [14, 13] which assumed an Hermitian form for the mass matrices with texture zeros in the (1, 1) and (1, 3), (3, 1) elements. Although any matrix can be made Hermitian by phase changes in general the texture zeros are not preserved by such transformations so this is the most general starting point.

To summarize we have shown that it is sufficient in a parameterization of the mass matrices to retain two non-zero phases which we take to be ϕ_{12}^U and ϕ_{13}^D , i.e. $\phi_1 = \phi_{12}^U$ and $\phi_2 = \phi_{13}^D$. Our analysis requires $\phi_{12}^U \approx 90^\circ$ which is the case of ‘maximal’ CP violation for fixed quark mass ratios (*c.f.* [14]).

3.2 Fit to the data: symmetric texture

We now turn to a fit to the data with a non-zero entry in the (1,3) position of the down quark mass matrix. As we have discussed the fit to CKM matrix elements is in this case insensitive to the matrix elements below the diagonal. For definiteness we perform a fit making the assumption that the mass matrix is symmetric. This corresponds to a specific choice for the elements below the diagonal consistent with this “smallness” criterion. Of course, assuming an Hermitian rather than a symmetric form for the mass matrices does not change the quality of the fit.

Leaving aside a discussion of phases for the moment, the U and D Yukawa matrices have the form

$$U/h_t = \begin{pmatrix} 0 & b'\epsilon^3 & c'\epsilon^4 \\ b'\epsilon^3 & \epsilon^2 & a'\epsilon^2 \\ c'\epsilon^4 & a'\epsilon^2 & 1 \end{pmatrix} \quad (24)$$

and

$$D/h_b = \begin{pmatrix} 0 & b\bar{\epsilon}^3 & c\bar{\epsilon}^4 \\ b\bar{\epsilon}^3 & \bar{\epsilon}^2 & a\bar{\epsilon}^2 \\ c\bar{\epsilon}^4 & a\bar{\epsilon}^2 & 1 \end{pmatrix} \quad (25)$$

where h_t, h_b are the t, b Yukawa couplings and the expansion parameters ϵ and $\bar{\epsilon}$ will be chosen so that the remaining parameters a, b, c, a', b' and c' are all of $O(1)$. This texture is similar to the *ansatz* considered by Branco et al.[29].

In fact we can extend this parameterization to include charged lepton masses by choosing the same matrix L (with the same parameters) for the charged leptons mass matrix as for the down quarks except for the usual Georgi-Jarlskog [30] factor of -3 multiplying the (2,2) element. This retains m_b - m_τ unification and also gives good predictions for the lighter generations of lepton.

The small expansion parameters are determined immediately, since to leading order (all our discussion is to leading order) we have

$$\frac{m_c}{m_t} = \epsilon^2 \quad \frac{m_s}{m_b} = \bar{\epsilon}^2 \quad (26)$$

Since the above texture should apply at the unification scale, we have

$$\epsilon \simeq 0.05 \quad \bar{\epsilon} \simeq 0.15 \quad (27)$$

i.e. we see that $\epsilon < \bar{\epsilon}$ and suggests $\epsilon = \mathcal{O}(\bar{\epsilon}^2)$. The coefficients b and b' are also determined to leading order since

$$\frac{m_u}{m_c} = (b'\epsilon)^2 \quad \frac{m_d}{m_s} = (b\bar{\epsilon})^2 \quad (28)$$

giving $b \simeq 1.5$ and $b' \simeq 1$. So far the parameters are taken real but we now introduce phases as discussed in Section 3.1. For the case of interest with a small (1,3) element only two phases play a role in determining the physics, as discussed above. Here we assign a phase to each of U and D by taking

$$b' \rightarrow b' e^{i\phi} \quad c \rightarrow c e^{i\psi} \quad (29)$$

Having chosen to attach a phase ϕ to the (1,2) element of U , as discussed in section 3.1 the one phase chosen in D should be attached to a different element of D . We then expand all the elements of the rotation matrices which diagonalize U and D in terms of ϵ and $\bar{\epsilon}$, retaining the leading terms only. The CKM elements are expressed in terms of these rotation matrix elements (see [8] for example) which then allow leading order expressions for the V_{ij}^{CKM} in terms of our expansion parameters.

At this leading order, we have

$$V_{us} = b\bar{\epsilon} - b'\epsilon e^{i\phi} \implies |V_{us}| = \lambda = \sqrt{\frac{m_d}{m_s}} (1 + \mathcal{O}(\epsilon/\bar{\epsilon})) \quad (30)$$

where λ is the Wolfenstein parameter. The Wolfenstein parameter A fixes the value of $|V_{cb}|$ and in our expansion

$$|V_{cb}| = A\lambda^2 = a\bar{\epsilon}^2 + \mathcal{O}(\epsilon\bar{\epsilon}^4) \implies A = \frac{a}{b^2}. \quad (31)$$

We are neglecting here the up quark contribution to $|V_{cb}|$, which is justified if the (2,3) elements in the up quark matrix are indeed of order ϵ^2 as suggested by eq. (24). From a phenomenological point of view, however, a larger size for those elements (e.g. of order ϵ) is also allowed and could lead to non negligible up quark contributions to V_{cb} .

Since the textures that we are discussing apply at the unification scale rather than at low energies, we must use values for mass ratios and CKM parameters appropriate to that scale. For this it is convenient to introduce the parameter $\chi = (M_X/M_Z)^{-h_i^2/(16\pi^2)} \approx 0.7$ and then

$$\begin{aligned} \frac{A(M_X)}{A(M_Z)} &= \chi \\ \frac{(m_s/m_b)(M_X)}{(m_s/m_b)(M_Z)} &= \chi \\ \frac{(m_c/m_t)(M_X)}{(m_c/m_t)(M_Z)} &= \chi^3 \end{aligned} \quad (32)$$

Thus at the unification scale we have $A \simeq 0.58$ or $a \simeq 1.3$. As discussed above, up to corrections suppressed by inverse powers of the third generation masses, the phase ϕ determines the sign and magnitude of the CP-violating CKM phase. In the SM context, the observed CP violation requires a near maximal phase, $\phi \approx 90^\circ$, so

$$\begin{aligned} V_{ub} &= c\bar{\epsilon}^4 e^{i\psi} - iab'\epsilon\bar{\epsilon}^2 \\ V_{td} &= -c\bar{\epsilon}^4 e^{i\psi} + ab\bar{\epsilon}^3 \end{aligned} \quad (33)$$

which imply that, to leading order the Wolfenstein parameters which govern the size of V_{ub} and V_{td} are given by

$$\begin{aligned}\rho &= \left(\frac{b'\epsilon}{b\bar{\epsilon}}\right)^2 + \frac{\bar{\epsilon}}{ab} c \cos \psi - \frac{b'\epsilon}{ab^2} c \sin \psi \\ \eta &= \frac{b'\epsilon}{b\bar{\epsilon}} - \frac{\bar{\epsilon}}{ab} c \sin \psi - \frac{b'\epsilon}{ab^2} c \cos \psi\end{aligned}\quad (34)$$

For c small, the phase $\phi \simeq +90^\circ$ fixes the correct sign of the first (dominant) term in the expression for η and maximizes CP violation for fixed quark mass ratios. In passing we note that, to this order, the entire list of quark masses and CKM matrix elements do not involve a' or c' and so could take on any value of $O(1)$ without affecting the physics. We can express the perturbation to the canonical values for $|V_{ub}/V_{cb}|$ and $|V_{td}/V_{ts}|$ given by eq(1)

$$\begin{aligned}\frac{|V_{ub}|^2}{|V_{cb}|^2} &= \frac{m_u}{m_c} \left(1 - 2\frac{c\bar{\epsilon}^2 \sin \psi}{ab'\epsilon} + \frac{c^2\bar{\epsilon}^4}{a^2b'^2\epsilon^2}\right) \\ \frac{|V_{td}|^2}{|V_{ts}|^2} &= \frac{m_d}{m_s} \left(1 - 2\frac{c\bar{\epsilon} \cos \psi}{ab} + \frac{c^2\bar{\epsilon}^2}{a^2b^2}\right)\end{aligned}\quad (35)$$

The effect of filling in the (1,3) texture zero of D can be more dramatic for $|V_{ub}/V_{cb}|$ than for $|V_{td}/V_{ts}|$ since the correction to the latter is suppressed by $\epsilon/\bar{\epsilon} \sim \sqrt{m_u/m_c}/\sqrt{m_d/m_s}$ relative to the correction to the former. We can get the desired phenomenological result of moving $|V_{ub}/V_{cb}|$ up towards the measured value around 0.09 while not unduly perturbing the value of $|V_{td}/V_{ts}|$ given by eq(1).

Parameter	Value	Referen.
Q	22.7 ± 0.8	[25]
m_u/m_d	0.533 ± 0.043	[25]
m_c/m_s	9.5 ± 1.7	[15]

Table 3: Values for the quark masses ratios and the parameter Q (defined in eq(36)) used for the texture zero fits.

Using the expansions of eqs(24,25) for the U and D matrices in terms of ϵ and $\bar{\epsilon}$ we carry out a fit using information on the measured estimates for ratios of quark masses and CKM matrix elements. The expansion parameters ϵ , $\bar{\epsilon}$ and the $O(1)$ coefficients a , b , b' and c are determined via the expressions (26,28,30,31,34,35). We follow BHR in using the combination Q given by

$$Q = \frac{m_s/m_d}{\sqrt{1 - (m_u/m_d)^2}} \quad (36)$$

which is determined accurately from chiral perturbation theory. Additionally we can use the ratios of the masses m_u/m_d , m_c/m_s .

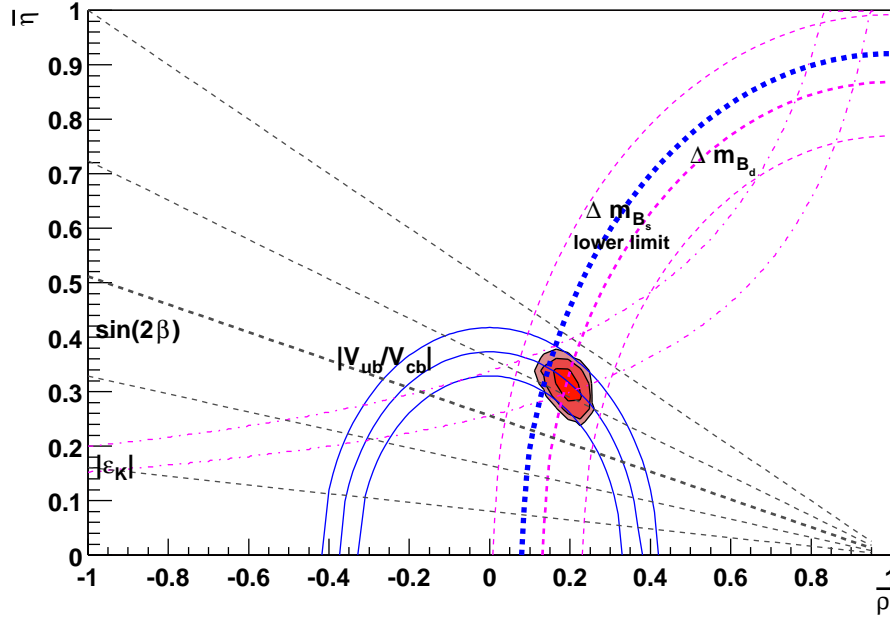


Figure 3: Fit A of Section 3.2 (symmetric texture) to the measurements of $|V_{ub}/V_{cb}|$, Δm_{B_s} , Δm_{B_d} , $|\epsilon_K|$ and $\sin 2\beta$.

The resulting fit (Fit A) yields the values

$$\begin{aligned} \bar{\epsilon} &= 0.15 \pm 0.01 & |b| &= 1.5 \pm 0.1 & a &= 1.31 \pm 0.14 \\ |c| &= 2.7 \pm 0.10 & \psi &= -24^\circ \pm 3^\circ \end{aligned} \quad (37)$$

where the values are given at the unification scale.

Demanding a texture zero in the (1,3),(3,1) elements for a symmetric D matrix leads, in particular, to too small a value for $\bar{\rho}$ and there is a marked improvement in the overall description of the data when this zero is filled in, as can be seen by comparing Fig. 3 with Fig. 2.

That $c \sim 3$ means that the order of the (1,3) term is ambiguous and could be either $O(\bar{\epsilon}^4)$ or $O(\bar{\epsilon}^3)$.³ While the texture zero in D_{13} , D_{31} does lead to the rather attractive result of eq(1) the current experimental data, within the context of the Standard Model, favour perturbative corrections as suggested by eqs(35).

We noted earlier (Section 2.1) that there is a potential disagreement in the SM fit to $|\epsilon_K|$, Δm_{B_s} , Δm_{B_d} and the recent measurement of $\sin 2\beta = 0.41 \pm 0.17$ [6]. To quantify the implications of this, we perform a second version of Fit A (still using the texture of eqs(24,25)), dropping the constraints of $|\epsilon_K|$, Δm_{B_s} and Δm_{B_d} . This is shown in Fig. 4. The effect is to increase slightly the value of the parameter c to 3.31 ± 0.10 and the value of ψ to $6^\circ \pm 3^\circ$.

In Fig. 5 we show the resulting probability distributions for the quantities $\frac{V_{ub}}{V_{cb}}$, $|\epsilon_K|$, Δm_{B_s} and $\sin 2\beta$ compared to the corresponding experimental distributions.

³That c' is quite undetermined means that a texture zero in U_{13} , U_{31} is not ruled out.

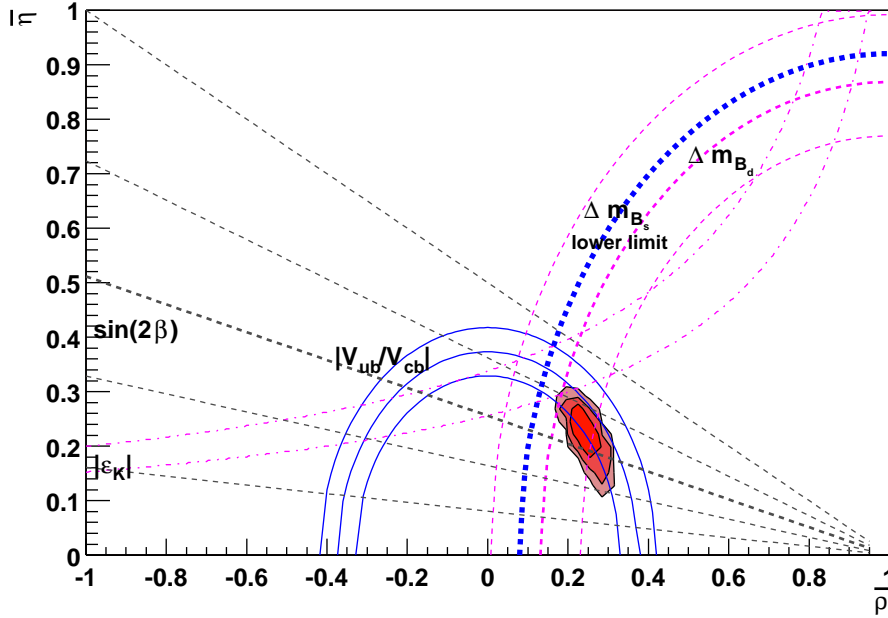


Figure 4: Fit A of section 3.2 (symmetric texture) to the measurements of $|V_{ub}/V_{cb}|$ and $\sin 2\beta$.

There is actually a further solution (Fit B) to the above equations where the phase $\phi \sim -90^\circ$ rather than $+90^\circ$ which corresponds to $b' \rightarrow -b'$ in eqs(34,35). In this case the first term in the expression for η is no longer the dominant one and a larger value of c is needed to obtain a positive value for η . Only the parameters c and ψ change from Fit A, and for Fit B we find $c = 8.45 \pm 0.33$, $\psi = -58^\circ \pm 5^\circ$. The result of Fit B is shown in Fig. 6 and comparing with Fig. 3 we see no difference in the quality of fits A and B. This solution has the (1,3) matrix element of the same order ($O(\bar{\epsilon}^3)$) as the (1,2) matrix element. In this case $D_{13}/D_{12} \approx D_{23}/D_{22} \approx 1$, suggestive of a non-Abelian family structure.

3.3 Fit to the data: asymmetric texture

We now consider a fit to the data in which the texture zero relation $|V_{ub}/V_{cb}| = \sqrt{m_u/m_c}$ is modified by higher order corrections in the perturbative diagonalization of the down mass matrix. As in the previous section, the correction to that relation comes from a small but non negligible rotation s_{13}^D induced by a non zero element \tilde{D}_{13} . Here, however, we assume that \tilde{D}_{13} is mainly induced by the rotation $s_{23}'^D$ used to diagonalize the 23 sector of the down quark mass matrix, the initial value D_{13} being negligible (see eq. (16)). This situation is therefore complementary to the one considered in the previous subsection, where \tilde{D}_{13} was mainly given by the original entry D_{13} and the contribution to \tilde{D}_{13} proportional to $s_{23}'^D$ was assumed to be negligible.

Let us first of all estimate how large $s_{23}'^D$ should be in order to give a significant contribution to $|V_{ub}/V_{cb}|$. We start from eqs. (18), that assume only $|Y_{12}| = |Y_{21}|$, $Y_{11} = 0$. The size of the

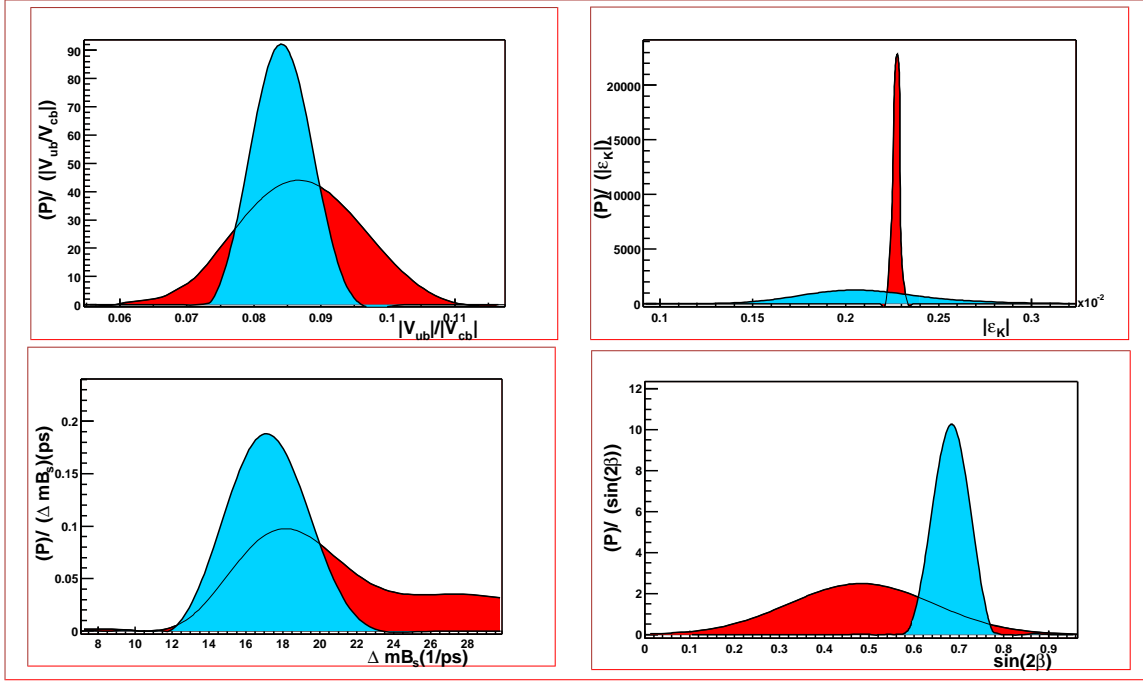


Figure 5: One dimensional probabilities for the physical observables $|V_{ub}|/|V_{cb}|$, $|\epsilon_K|$, Δm_{B_d} and $\sin 2\beta$. The probabilities in red (dark) correspond to the experimental constraints and the probabilities in blue (light) correspond to the predictions of the texture with 13 and 31 entries different from zero with $c = (2.7 \pm 0.10)$ and $\psi = (-24 \pm 3)^0$.

correction is determined by s_{13}^Y , which can be written as

$$s_{13}^Y = \frac{\tilde{Y}_{13}}{Y_{33}} \sim \sqrt{\frac{m_1}{m_2} \frac{m_2}{m_3}} s_{23}^Y, \quad (38)$$

where m_i is the mass of the quark of the i -th family in the sector $Y = U, D$ and we have used $\tilde{Y}_{13} = Y_{12} s_{23}^Y$, $Y_{12}/\tilde{Y}_{22} \sim \sqrt{m_1/m_2}$ and $\tilde{Y}_{22}/Y_{33} \simeq m_2/m_3$. From eq. (38) one can see that the contribution of the up quark rotation s_{13}^U to $s_{13} = s_{13}^D - s_{13}^U$ is negligible. The mass ratios in eq. (38) are in fact much smaller in the up sector than in the down sector. Moreover, the factor s_{23}^Y in (38) appears in the product $s_{23}^Y s_{23}^Y$, which contributes to the ratio m_2/m_3 . Barring cancellations, we therefore have $s_{23}^Y s_{23}^Y \lesssim m_2/m_3$, a constraint stronger in the up sector due again to $m_c/m_t \ll m_s/m_b$. Hence we can safely neglect s_{13}^U and write

$$\frac{s_{13}}{s_{23}} \simeq \frac{s_{13}^D}{s_{23}} \sim \sqrt{\frac{m_d}{m_s} \frac{m_s}{m_b}} \frac{s_{13}^D}{|V_{cb}|}. \quad (39)$$

Since at the electroweak scale we have $\sqrt{m_d/m_s} \sim 0.22$ and $(m_s/m_b)/|V_{cb}| \sim 0.6$, in order to get a correction $s_{13}/s_{23} \sim 0.03$ (which gives a good agreement with data in absence of phases) it is sufficient to have $s_{13}^D \sim 0.2 - 0.3$, or $|D_{32}| \sim (0.2 - 0.3)|D_{33}|$. Larger values are also in principle allowed depending on the phases in eq. (19). Notice that in first approximation the ratio D_{32}/D_{33} at the unification scale is the same as at the electroweak scale.

Quark mass textures with $|D_{32}/D_{33}| = \mathcal{O}(1)$ have been considered in the literature [4] in connection with a large leptonic mixing angle originating from the charged lepton mass matrix.

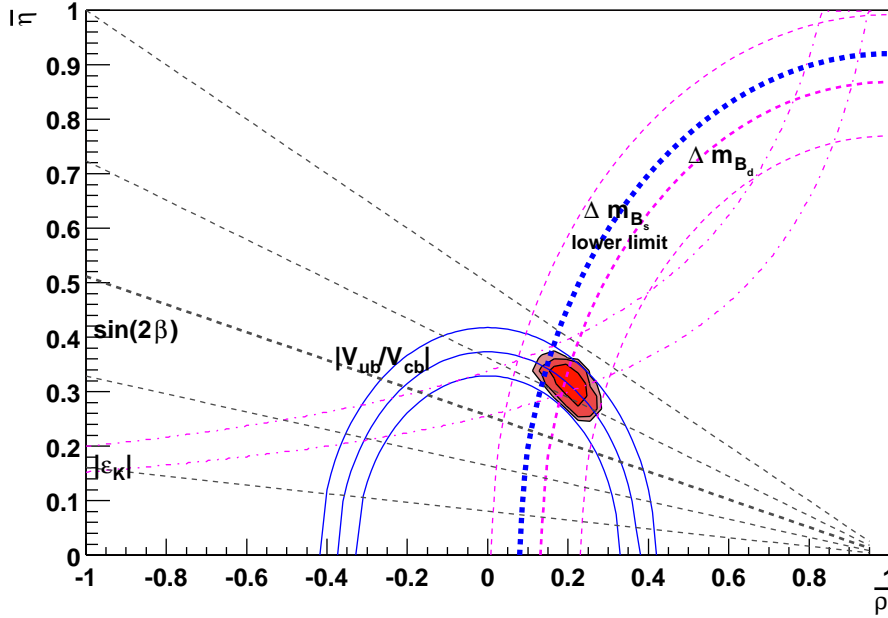


Figure 6: Fit B of Section 3.2 (symmetric texture) to the measurements of $|V_{ub}/V_{cb}|$, Δm_{B_s} , Δm_{B_d} , $|\epsilon_K|$ and $\sin 2\beta$.

In SU(5) unified models, the ratio $|D_{32}/D_{33}|$ associated with the right-handed quark rotation s'^D_{13} corresponds in fact to the ratio of charged lepton mass matrix elements $|E_{23}/E_{33}|$ associated with a left-handed charged lepton rotation that mixes μ and τ neutrinos. On one hand, this link can be considered as a motivation for studying asymmetric textures as a solution of the V_{ub}/V_{cb} problem. From the opposite point of view, we can say that in the context of unified models, the asymmetric solution of the V_{ub}/V_{cb} problem, pointing at a largish value of $|D_{32}/D_{33}|$, indicates that the large mixing angle responsible for the atmospheric neutrino anomaly comes, or at least receives a significant contribution, from the diagonalization of the charged lepton mass matrix.

The precise relation between the right-handed quark and left-handed lepton rotations depends on possible Clebsch coefficients relating transposed down quark and charged lepton matrix elements. The presence of non-trivial coefficients enhancing some charged lepton matrix element is indeed suggested by the empirical relation $m_\mu/m_\tau \sim 3m_s/m_b$. Since a value of $|D_{32}/D_{33}|$ around 1/3 is preferred by a fit of data, a near to maximal lepton mixing can be obtained if the Georgi-Jarskog factor 3 sits in the E_{23} entry.

Notice that a sizeable $|D_{32}| \sim |D_{33}|/3$ indicates an asymmetry $|D_{32}| \gg |D_{23}|$. The ratio $|D_{23}/D_{33}|$ is in fact expected to be of order $|V_{cb}| \ll 1/3$, barring cancellations between up and down quark contributions to V_{cb} . Such an asymmetry can be easily obtained in the context of Abelian and non-Abelian models. In Section 4.2 we will describe an explicit example of non-Abelian family symmetry leading to the asymmetry in the 23 sector while preserving the relation $|Y_{12}| = |Y_{21}|$ in the 12 sector and the texture zeros $Y_{13} \simeq Y_{31} \simeq Y_{11} \simeq 0$. This texture allows us to isolate and study the corrections to the texture zero relations we are considering in this section.

To examine the implications of this scenario we modify the parameterisation of the Yukawa matrices given in eqs(24, 25) by restoring the exact texture zeros in the (1,3) and (3,1) elements and allow $|D_{32}| \gg |D_{23}|$.

We consider the following parameterisations for the mass matrices for the up and down quarks whose absolute values can be parametrized as

$$|U/h_t| = \begin{pmatrix} 0 & c\epsilon\epsilon' & 0 \\ c\epsilon\epsilon' & \beta\epsilon^2 & b\epsilon \\ 0 & a\epsilon & 1 \end{pmatrix} \quad (40)$$

$$|D/h_b| = \begin{pmatrix} 0 & \epsilon' & 0 \\ \epsilon' & \alpha\epsilon & \epsilon \\ 0 & t & 1 \end{pmatrix}, \quad (41)$$

with $\epsilon' < \epsilon \ll 1$. In this parameterisation of D we see, comparing with eq(25), we have changed the (3,2) element to be $O(1)$. The remaining elements are of the same order, ϵ in eq(41) being $O(\bar{\epsilon}^2)$ and ϵ' is $O(\epsilon^3)$. The parameterisation of U has the same form as eq(24) with the exception of the (2,3) and (3,2) elements which are now of $O(\epsilon)$ and not $O(\epsilon^2)$. We have chosen this form to relate to a promising texture model discussed in Section 4.2. However these elements are poorly determined by the data and it is possible to obtain solutions where a, b are $O(\epsilon)$, corresponding to the original symmetric parametrisation of eq(24).

In order to obtain the precise form of the corrected texture zero relations, we first write the general expression for $|V_{us}|$, $|V_{ub}/V_{cb}|$, $|V_{td}/V_{ts}|$ in terms of the rotations defined by eq. (9):

$$\begin{aligned} |V_{us}| &= |t_{12}^D - t_{12}^U e^{i\phi_1}| c_{12}^D \\ \left| \frac{V_{ub}}{V_{cb}} \right| &= \left| t_{12}^U - \frac{s_{13}}{s_{23}} e^{i(\phi_2 - \phi_1)} \right| \\ \left| \frac{V_{td}}{V_{ts}} \right| &= \left| t_{12}^D - \frac{s_{13}}{s_{23}} e^{i\phi_2} \right|, \end{aligned} \quad (42)$$

where t_{12}^D and t_{12}^U are the tangent of the 12 rotation of left-handed down and up quark respectively. Written as above in terms of the tangent of the angles (and cosines c_{12}^D , c_{12}^U), the expressions are exact up to $\mathcal{O}(\lambda^4)$ corrections, λ being one of the Wolfenstein parameters. The phases ϕ_1 , ϕ_2 were discussed in Section 3.1. In order to obtain the relation between mixing angles and quark mass ratios generalizing the texture zero relations, one has to express the angles in eqs. (42) in terms of quark masses. In the case of textures (41,40), we obtain

$$\begin{aligned} t_{12}^U &= \sqrt{\frac{m_u}{m_c}} \\ t_{12}^D &= \sqrt{c \frac{m_d}{m_s}} \left(1 - \frac{1}{2} t^2 c \frac{m_d}{m_s} \right) \\ \frac{s_{13}}{s_{23}} &= t \sqrt{c \frac{m_d}{m_s} \frac{m_s/m_b}{|V_{cb}|}}, \end{aligned} \quad (43)$$

where $t = |D_{32}/D_{33}|$ as in eq. (41). The parameter t represents the tangent t'_{23} of the 23 rotation on the right-handed down quarks in eq. (9). Since we are considering the possibility of a sizeable t , we do not approximate the cosine of that angle, $c \equiv 1/\sqrt{1+t^2}$, with 1. As a consequence we have, at leading order in $\lambda^2 = \mathcal{O}(m_d/m_s)$,

$$t_{12}^D \simeq \sqrt{c \frac{m_d}{m_s}} \neq \sqrt{\frac{1}{c} \frac{m_d}{m_s}} \simeq t'_{12}^D.$$

This is because the diagonalization of the 23 sector in the down sector not only induces an effective (1,3) element, but also generates a slight asymmetry in the 12 sector: $|\tilde{D}_{12}| \neq |\tilde{D}_{21}|$ despite $|D_{12}| \neq |D_{21}|$. The expression for t_{12}^D above also includes a next to leading correction in $\lambda^2 = \mathcal{O}(m_d/m_s)$.

Let us now turn to the numerical determination of the parameters entering the expressions for U and D . Three of those parameters, t and the two phases ϕ_1, ϕ_2 , can be determined independently of the values of the others in terms of $|V_{us}|$ and $\bar{\rho}, \bar{\eta}$ (as obtained from the SM fit). This is possible since, for given values of the quark masses, eqs. (42,43) relate t, ϕ_1, ϕ_2 to $|V_{us}|$ and $|V_{ub}/V_{cb}|, |V_{td}/V_{ts}|$, and therefore to $|V_{us}|$ and $\bar{\rho}, \bar{\eta}$. More precisely, to get the best values of t, ϕ_1, ϕ_2 , we use the following procedure. First we calculate $|V_{ub}/V_{cb}|, |V_{td}/V_{ts}|$ in terms of $\bar{\rho}, \bar{\eta}$. Then, for any given value of the ratio $t = |D_{32}/D_{33}|$ and of the phase⁴ ϕ_2 we recover m_d/m_s and m_u/m_c from eqs. (42,43) (the phase ϕ_1 is obtained, up to discrete ambiguities, from the relation involving $|V_{us}|$). Finally, we calculate $Q, m_u/m_d$ in terms of $m_d/m_s, m_u/m_c$. We can at this point perform a fit of $Q, m_u/m_d$ in terms of $\bar{\rho}, \bar{\eta}$ for any given value of t and ϕ_2 . The quantities $m_c/m_s, m_s/m_b, |V_{cb}|, \lambda$ involved in the relation between $Q, m_u/m_d$ and $\bar{\rho}, \bar{\eta}$ are also included in the fit. As mentioned before, using $Q, m_u/m_d$ instead of $m_d/m_s, m_u/m_c$ improves considerably the quality of the fit, especially for small t (despite it requiring the inclusion of the ratio m_c/m_s) [11]. We then obtain $t = 0.3, \phi_2 \simeq -0.2\pi, \cos \phi_1 \simeq 0.1$. Notice that ϕ_1 again turns out to be almost maximal as was the case with symmetric textures.

The determination of t, ϕ_1, ϕ_2 described above depends on the additional parameters in (40,41) only through the quark masses and $|V_{cb}|$. As a consequence, the fit of all data in terms of all parameters decouples into a fit of $|V_{us}|, \bar{\rho}, \bar{\eta}$ in terms of t, ϕ_1, ϕ_2 (which makes use of the experimental values of the quark masses and $|V_{cb}|$) and a fit of quark masses and $|V_{cb}|$ in terms of the additional parameters. The first fit has been described above and is independent of the specific form of the textures (41,40) and of the values of the additional parameters one uses to account for the quark masses and $|V_{cb}|$. For example, independently of whether the up quark matrix is fully symmetric or not, the parameters a and b are of $\mathcal{O}(1)$ or smaller, α, β are of $\mathcal{O}(1)$ or vanishing, provided that the values of the quark masses and of $|V_{cb}|$ can be accounted for. All the dependence on the specific form of (40,41) is therefore confined to the fit of quark masses in terms of the parameters $\epsilon, \epsilon', a, b, c, \alpha, \beta$. Here we consider in more detail such a fit in the case motivated by the flavour model described in Section 4.2 in which $\alpha = \beta = 0$.

⁴To be precise, in order to include all sign ambiguities in a single phase we actually fix the value of the phase ϕ'_2 defined by $\cos \phi'_2 = \cos \phi_2, \sin \phi'_2 = \sin \phi_2 \text{sign}(\sin \phi_1)$. In the following, ϕ_2 should be read ϕ'_2 .

Let us start from the quark masses. Besides m_t , m_b , trivially accounted for by h_t , h_b , we have to account for m_s/m_b , m_c/m_t , $m_d m_s/m_b^2$, $m_u m_c/m_t^2$. We work at the m_t scale. In order to account for $m_s/m_b \simeq t\epsilon$ we need $\epsilon \simeq 0.08$. From $m_c/m_t \simeq ab\epsilon^2$ we then have $ab \simeq 0.6$. Since $m_d m_s/m_b^2 = \epsilon'^2/(1+t^2)^{3/2}$, we can obtain $\epsilon' = 0.006$. The ratio $m_u m_c/m_t^2 = (c\epsilon\epsilon')^2$ then gives $c \simeq 0.5$. Notice that all parameters that are supposed to be of $O(1)$ indeed are. In particular, we have $|U_{23}/h_t| \sim |U_{32}/h_t| \sim |D_{23}/h_b|$, which justifies the use of the same parameter (ϵ) in both the U and D matrices. In the model of Section 4.2 the same parameter indeed appears in those entries because the same vev generates them. Finally, we have to account for the value of $|V_{cb}|$, which has a contribution from the down sector, ϵ , and one from the up sector, $b\epsilon$. Both contributions are of the right order of magnitude, so it is clear that $|V_{cb}|$ can be obtained for an appropriate choice of the $O(1)$ coefficient b . The precise value of b depends on the relative phase between the two contributions. For $\alpha = \beta = 0$, the phases of the SM quark multiplets can be redefined in such a way that U/h_t and D/h_b differ from their absolute values in (41,40) only by a phase $e^{i\phi}$ multiplying t in D_{32} and a phase $e^{i\psi}$ multiplying b in U_{23} . The phases ϕ_1 and ϕ_2 are then given by

$$e^{i\phi_1} = e^{i(\psi-\phi)} \quad (44)$$

$$e^{i\phi_2} = \frac{\epsilon(b e^{i\psi} - 1)}{|V_{cb}|}. \quad (45)$$

The relative phase between the two contributions to V_{cb} turns out to be $e^{i\psi}$ and can be determined from the equations above. We are actually interested to the value of b which simply follows from eq. (45): $b = |e^{i\phi_2}|V_{cb}|/\epsilon + 1| \simeq 1.4$. We then also have $a \simeq 0.4$. Notice that a and b are both of $O(1)$ but $b > a$. This slight asymmetry is reduced by the running up to the unification scale.

The results for this fit with the asymmetric texture, $D_{32} \gg D_{23}$ are very close to those for the symmetric texture of 3.2 and the resulting contour plot in the $\bar{\rho} - \bar{\eta}$ plane is essentially identical to that in fig. 3.

4 Implications for a family symmetry

Of course the underlying motivation for studying the detailed structure of the quark and lepton mass matrices is that they may lead to an insight about the structure beyond the Standard Model. Here we briefly comment on the implications of our analysis for such structure concentrating on the possibility there is an extension of the symmetries of the Standard Model to include a family symmetry.

4.1 Symmetric case

It turns out to be remarkably easy to construct a model leading to the mass matrices in eqs. (24,25) through the introduction of an Abelian gauge symmetry, $U(1)$ (such additional

	Q_i	u_i^c	d_i^c	L_i	e_i^c	ν_i^c	H_2	H_1
$U(1)_{FD}$	α_i	α_i	α_i	a_i	a_i	a_i	$-2\alpha_1$	$-2\alpha_1$

Table 4: $U(1)_{FD}$ symmetries.

symmetries abound in string theories). The most general charge assignment of the Standard Model states is given in Table 4. This follows since the need to preserve $SU(2)_L$ invariance requires (left-handed) up and down quarks (leptons) to have the same charge. This together with the requirement of symmetric matrices then requires that all quarks (leptons) of the same i -th generation transform with the same charge $\alpha_i(a_i)$. If the light Higgs, H_2 , H_1 , responsible for the up and down quark masses respectively have $U(1)$ charge so that only the (3,3) renormalisable Yukawa coupling to H_2 , H_1 is allowed, only the (3,3) element of the associated mass matrix will be non-zero as desired. The remaining entries are generated when the $U(1)$ symmetry is broken. A particularly interesting example may be constructed in a supersymmetric extension of the Standard Model [31]. We assume this breaking is spontaneous via Standard Model singlet fields, θ , $\bar{\theta}$, with $U(1)_{FD}$ charge -1, +1 respectively, which acquire vacuum expectation values (vevs), $\langle \theta \rangle$, $\langle \bar{\theta} \rangle$, along a “D-flat” direction. After this breaking all entries in the mass matrix become non-zero. For example, the (3,2) entry in the up quark mass matrix appears at $O(\epsilon^{|\alpha_2-\alpha_1|})$ because $U(1)$ charge conservation allows only a coupling $c^t H_2(\theta/M_2)^{\alpha_2-\alpha_1}$, $\alpha_2 > \alpha_1$ or $c^t H_2(\bar{\theta}/M_2)^{\alpha_1-\alpha_2}$, $\alpha_1 > \alpha_2$ and we have defined $\epsilon = (\langle \theta \rangle / M_2)$ where M_2 is the unification mass scale which governs the higher dimension operators. As discussed in reference [31] one may expect a different scale, M_1 for the down quark mass matrices (it corresponds to mixing in the H_2 , H_1 sector with M_2 , M_1 the masses of heavy H_2 , H_1 fields). Thus we arrive at mass matrices of the form

$$\frac{M_u}{m_t} \approx \begin{pmatrix} h_{11}\rho_{11}\epsilon_a^{|2+6a|} & h_{12}\rho_{12}\epsilon_b^{|3a|} & h_{13}\rho_{13}\epsilon_a^{|1+3a|} \\ h_{21}\rho_{21}\epsilon_b^{|3a|} & h_{22}\rho_{22}\epsilon^2 & h_{23}\rho_{23}\epsilon^1 \\ h_{31}\rho_{31}\epsilon_a^{|1+3a|} & h_{32}\rho_{32}\epsilon^1 & h_{33} \end{pmatrix} \quad (46)$$

$$\frac{M_d}{m_b} \approx \begin{pmatrix} k_{11}\sigma_{11}\bar{\epsilon}_a^{|2+6a|} & k_{12}\sigma_{12}\bar{\epsilon}_b^{|3a|} & k_{13}\sigma_{13}\bar{\epsilon}_a^{|1+3a|} \\ k_{21}\sigma_{21}\bar{\epsilon}_b^{|3a|} & k_{22}\sigma_{22}\bar{\epsilon}^2 & k_{23}\sigma_{23}\bar{\epsilon}^1 \\ k_{31}\sigma_{31}\bar{\epsilon}_a^{|1+3a|} & k_{32}\sigma_{32}\bar{\epsilon}^1 & k_{33} \end{pmatrix} \quad (47)$$

where $\bar{\epsilon} = (\frac{\langle \bar{\theta} \rangle}{M_1})^{|\alpha_2-\alpha_1|}$, $\epsilon = (\frac{\langle \theta \rangle}{M_2})^{|\alpha_2-\alpha_1|}$, and $a = (2\alpha_1 - \alpha_2 - \alpha_3)/3(\alpha_2 - \alpha_1)$. For $-3a > 1$ $\epsilon_a = \epsilon_b = \epsilon$ and $\bar{\epsilon}_a = \bar{\epsilon}_b = \bar{\epsilon}$. In this case it is easy to check that there are *no* texture zeros because all matrix elements contribute at leading order to the masses and mixing angles. For $1 > -3a > 0$, ϵ_a , $\bar{\epsilon}_a$ change and are given by $\bar{\epsilon}_a = (\frac{\langle \bar{\theta} \rangle}{M_1})^{|\alpha_2-\alpha_1|}$, $\epsilon_a = (\frac{\langle \theta \rangle}{M_2})^{|\alpha_2-\alpha_1|}$. In this case texture zeros in the (1,1) and (1,3) positions *automatically* appear for small $\langle \bar{\theta} \rangle$. However the (1,2) matrix element is too large (cf Table 4). For $a > 0$ however $\bar{\epsilon}_{a,b} = (\frac{\langle \bar{\theta} \rangle}{M_1})^{|\alpha_2-\alpha_1|}$,

$\epsilon_{a,b} = (\frac{\langle \bar{\theta} \rangle}{M_2})^{|\alpha_2 - \alpha_1|}$, the texture zeros in the (1,1) and (1,3) positions persist, and the (1,2) matrix element can be of the correct magnitude.

Note that the family symmetry does not make the small elements exactly zero so it predicts only approximate texture zeros. Indeed, fixing the parameter $a = 1$ to obtain the measured magnitude of the (1,2) matrix element one finds that the (1,1) elements occurs at $O(\epsilon^8)$ and $O(\bar{\epsilon}^8)$ for the up and down mass matrices respectively. This is so small that eq(2) is valid to a high degree of accuracy if $\rho_{12} = \rho_{21}$. On the other hand the (1,3) matrix element is predicted to occur at $O(\epsilon^4)$, $O(\bar{\epsilon}^4)$ for the up and down matrices respectively and, as discussed above, a term of this order (with a coefficient 2) is sufficient to correct the prediction for $|\frac{V_{ub}}{V_{cb}}|$ following from the assumption of an exact texture zero.

However the best fit prefers the (1,3) element to occur at $O(\epsilon^3)$, $O(\bar{\epsilon}^3)$, i.e. close to the (1,2) matrix elements. Moreover the measured value of V_{cb} requires the (2,2) and (2,3) matrix elements of the down quark mass matrices should be of the same magnitude, of $O(\bar{\epsilon}^2)$. This is in contradiction to the predictions of the Abelian family symmetry, unless one appeals to the unknown coefficients of $O(1)$. The most plausible way to get such relations for matrix elements involving different family members is to invoke a non-Abelian family symmetry [5].

4.2 Asymmetric case

We now describe a supersymmetric non-Abelian model based on a $U(2)$ family symmetry acting on the two lighter families [27] and on the unified gauge group $SU(5)$. This model is a variation [32] which leads to asymmetric textures discussed in Section 3.3 with $Y_{22} \sim 0$. The lighter families ψ_a , $a = 1, 2$ ($\psi = T, \bar{F}$, where T and \bar{F} are respectively the **10** and $\bar{\mathbf{5}}$ representations of $SU(5)$) transform as $\psi_a \rightarrow U_{ab}\psi_b$ under $U \in U(2)$, whereas the third family and the Higgs fields H_1, H_2 are invariant. Such a symmetry is approximately realized in nature. In fact, in the $U(2)$ symmetric limit the lighter fermion families are forced to be massless and in supersymmetric models their scalar partners are forced to be degenerate. The symmetry is broken by two SM singlet scalars, an antidoublet ϕ^a transforming with U^{T-1} and an antisymmetric tensor A^{ab} transforming with $U^{T-1} \otimes U^{T-1}$ under $U(2)$. In an appropriate basis in the flavour space, the corresponding vevs can be written in the form

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ V \end{pmatrix} \quad \langle A \rangle = \begin{pmatrix} 0 & v \\ -v & 0 \end{pmatrix}, \quad (48)$$

where $V, v > 0$. The correct hierarchy and mixing between the two lighter families is obtained if $v/V = \mathcal{O}(|V_{us}|)$. The $U(2)$ breaking is communicated to the light fermions by an heavy $U(2)$ anti-doublet χ^a through a Froggatt-Nielsen mechanism. Under the gauge group, each χ^a , $a = 1, 2$, transforms as a full fermion family, which allows a mixing with the light fermions. The heavy mass term $M\chi^a\bar{\chi}_a$ for the fields χ^a also involves of course a doublet $\bar{\chi}_a$ with conjugated transformations under the SM and $U(2)$ group. As for the size of the mass term, one simple possibility is that the scale M is above the $SU(5)$ breaking scale, $M > M_{\text{GUT}}$. A small ratio

V/M is then generated if the $U(2)$ breaking takes place at the $SU(5)$ breaking scale, $V \sim M_{\text{GUT}}$. Possible $SU(5)$ breaking corrections to the heavy mass M will also be correspondingly smaller. The small ratio V/M determines the small Yukawa couplings accounting for the second generation masses and mixings, forbidden in the $U(2)$ symmetric limit. In particular, the correct order of magnitude for the mixing of the two heavier families is obtained if $V/M \sim \mathcal{O}(|V_{cb}|)$.

Besides $\chi^a, \bar{\chi}_a$, representing the minimal choice for the messenger sector, the physics at the GUT scale can involve additional heavy fields. For example, we have mentioned in the previous Section the possibility of a mixing in Higgs sector involving two heavy Higgs fields H'_1, H'_2 , singlets in this case under the $U(2)$ symmetry as H_1, H_2 . Such a mixing can be used to account for the hierarchy $m_b \ll m_t$. We therefore include H'_1, H'_2 in the model. Since they are allowed to interact both with the light families and the $U(2)$ breaking sector, the $U(2)$ singlets H'_1, H'_2 can also mediate $U(2)$ breaking. Notice that the scale M' at which this singlet-mediation takes place is a priori independent of the scale M associated to the doublet-mediation. For example, if the mass of the heavy Higgses is set by $SU(5)$ breaking we will have $M' \ll M$.

At this point one can write the most general renormalizable superpotential involving the light fermions (ψ_a, ψ_3), the Higgs fields (H_1, H_2), the $U(2)$ breaking fields (ϕ^a, A^{ab}) and the doublet ($\chi^a, \bar{\chi}_a$) and singlet (H'_1, H'_2) messengers. Once $U(2)$ is broken, a mixing between the previously massless fermions and the heavy messengers is generated. The new light fermions can then be easily identified by diagonalizing the heavy mass matrix. This leads to the following textures for the up and down quark mass matrices:

$$D/h_b = \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & 0 & \epsilon \\ 0 & t & 1 \end{pmatrix} \quad (49)$$

$$U/h_t = \begin{pmatrix} 0 & c\epsilon\epsilon' & 0 \\ -c\epsilon\epsilon' & 0 & b\epsilon \\ 0 & a\epsilon & 1 \end{pmatrix}, \quad (50)$$

where $\epsilon = \mathcal{O}(V/M)$, $\epsilon' = \mathcal{O}(v/M)$, $t = \mathcal{O}(V/M')$ and all other coefficients arise from couplings of order one. One then obtains the textures (41,40) for the absolute values of the mass matrices with $\alpha \sim \beta \sim 0^5$. In particular, the relation $|D_{12}| = |D_{21}|$ follows from the symmetry properties of the $U(2)$ representations⁶.

A few comments are in order. Since we expect $|D_{23}/D_{33}| = \mathcal{O}(|V_{cb}|)$, the texture zero in the (2,2) position requires $t = |D_{32}/D_{33}| \gg |V_{cb}|$ in order to account for the value of $|D_{32}/D_{33} \cdot D_{23}/D_{33}| \simeq m_s/m_b = \mathcal{O}(|V_{cb}|)$. We therefore expect a sizeable $t \gg \epsilon$, which leads to a non negligible correction to $|V_{ub}/V_{cb}|$. From the model building point of view, the asymmetry $|D_{32}| \gg |D_{23}|$ corresponding to $t \gg \epsilon$ can be simply accounted for by a relatively light singlet messenger scale $M' \ll M$ (which is analogous to the $M_1 \ll M_2$ assumption of the Abelian

⁵Unlike in eqs. (41,40), here the $\mathcal{O}(1)$ coefficients can be complex.

⁶A non negligible correction to $|D_{12}| = |D_{21}|$ can arise if $t = \mathcal{O}(1)$ from the diagonalization of the kinetic term.

case). In fact, if this is the case, the leading contribution to D_{32} comes from the exchange of the U(2) singlets H'_1, H'_2 at the scale M' . As a consequence, $|D_{32}|$ turns out to be larger than $|D_{23}|$, which is generated by the exchange of the U(2) doublets $\chi^a, \bar{\chi}_a$ at the higher scale M . Moreover, H'_1, H'_2 transform as **5** and $\bar{\mathbf{5}}$ of SU(5). Therefore, the singlet exchange at the lower scale M' does not contribute neither to the (2,3) nor to the (3,2) element in the up quark mass matrix, so that both U_{23} and U_{32} are of order ϵ . The larger hierarchy $m_c/m_t \ll m_s/m_b$ follows. As for the further suppression of $m_u m_c/m_t^2$ with respect to $m_d m_s/m_b^2$, that is automatically achieved if A^{ab} is a SU(5) singlet. The operator $A^{ab} T_a T_b H$ vanishes in fact in this case due to the antisymmetry of A^{ab} . This is a generic appealing feature of U(2) models. In order to generate a non-vanishing U_{12} entry, SU(5) breaking effects must be included either in the messenger masses or through higher dimension operators, thus giving the extra ϵ in U_{12} .

5 Conclusions

The presence of texture zeros in the quark Yukawa matrices can constrain quite tightly the detailed features of the CKM unitarity triangle. Recent data has shown that it is no longer viable for s_{13} to be zero and are, as a result, inconsistent with the most promising texture zero structure. This result seems quite reliable, following both from the improved bound on Δm_{Bs} as well as the improved value of $|V_{ub}/V_{cb}|$.

Theories invoking family symmetries beyond those of the standard model can lead to a hierarchal structure for U and D in which the elements appear in the form ϵ^k , where ϵ is a small parameter. Motivated by such an expansion, we have explored a perturbative approach in which the rotation s_{13} is small but non-zero. One way to do this is by allowing small entries to replace some of the texture zeros. We have investigated a common symmetric form for U and D where, in particular, the (1,3), (3,1) element is no longer zero. We have also investigated an alternative possibility for generating s_{13} by allowing an asymmetric form for the (2,3), (3,2) mass matrix elements. Both cases lead to a desirable phenomenological result whereby the perturbation of the ratio $|V_{ub}/V_{cb}|$ from the value $\sqrt{m_u/m_c}$ is larger than that of $|V_{td}/V_{ts}|$ from $\sqrt{m_d/m_s}$.

On the theoretical side, we would hope that pinning down the allowed structures for the Yukawa matrices will provide clues to the nature of an underlying family symmetry. Our analysis shows that a perturbative expansion in terms a small parameter is quite successful and therefore supports the idea that the family symmetry is spontaneously broken at the high energy scale. This has a concrete realization in the Froggatt Nielsen mechanism where light and heavy states are mixed via an extension of the ‘see-saw’ mechanism. If we require the symmetric form of the mass matrices it is necessary to have non-vanishing (1,3), (3,1) matrix elements. This is an interesting result because it follows from specific Abelian (and non-Abelian) family symmetries. Similarly the asymmetric solution can also be obtained from non-Abelian family symmetries. Improvements in the measurements of the quark masses and CKM mixing angles

and in particular on $\sin 2\beta$ (as well as on $\sin 2\alpha$) should help in distinguishing between these candidate symmetries and possibly lead to a viable theory of fermion mass generation.

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